DIMENSION OF FURSTENBERG MEASURES FOR $SL(2, \mathbb{C})$ PRODUCTS

Abstract. Let $\mathcal{A} = \{A_i\}_{i \in \Lambda}$ be a finite subset of $\mathrm{SL}(2, \mathbb{C})$, and let $p = (p_i)_{i \in \Lambda}$ be a probability vector with positive entries. Set $\theta := \sum_{i \in \Lambda} p_i \delta_{A_i}$, and denote by $\mathbf{S}_{\mathcal{A}}$ the semigroup generated by $\mathcal{A}$. Suppose that $\mathbf{S}_{\mathcal{A}}$ is strongly irreducible and proximal, and let μ be the Furstenberg measure on $\mathbb{CP}^1$ associated to θ.

We are working towards establishing the following result: If $\mathcal{A}$ is exponentially separated and no generalized circle $C \subset \mathbb{C}$ is invariant under the action of $\mathbf{S}_{\mathcal{A}}$ via Möbius transformations, then $\dim \mu = \min \{2, H(p)/(2\chi(\theta))\}$. Here, $H(p)$ is the entropy of p, and $\chi(\theta)$ is the Lyapunov exponent associated to θ.

Our approach relies on methods from additive combinatorics and involves an analysis of orthogonal projections of μ (viewed as a measure on $\mathbb{R}^2$). This is joint work with Ariel Rapaport.

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